Grating, Moire, and Speckle Methods for Measuring Displacement, Shape, Slope, and Curvature

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Displacement and Strain Measurement Using Gratings
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The Need For Higher Density Grating Leads to the Invention of the Moiré Method
Field Equation for In-plane Moiré Fringes

Recording the grating before and after deformation. Two fringes pattern are needed for strain analysis

\[ U = Np, \; \quad V = N'p \]

\( P \): Pith of the grating, or grating constant
\( N, N' = 0,1,2,3... \) fringes orders
Strain Measurement Using Moiré Fringes

Displacement and Strain Measurement Using Moire

0.0508 mm/fringe
Crack Tip Deformation
Displacement and Strain Measurement Using Moiré Fringes

Moiré fringe pattern with crossed gratings of different pitch on the master and the specimen

Shadow Moiré and Projection Moiré

Shadow- Moiré method with point illumination and point receiving

\[ w = \frac{Np}{\tan \alpha + \tan \beta}; \text{ when } \beta = 0 \]

\[ w = \frac{Np}{\tan \alpha} \]
Shadow Moiré for Measuring Shape

The Moiré Topogram of a scoliotic patient

X ray image

Normal

Takasaki

Deflection and Depth Moiré Fringes

Shadow Moiré

Moving-grating fringes

Projection Moiré

Stationary-grating fringes
Laser Speckle Photography

Laser Light
Optically Rough Surface
Recording of a laser speckle pattern

Laser Speckle Photography
White Light
Surface with attached or natural speckles
Recording of a white light speckle pattern
Speckle Methods for Displacement, Shape, and Slope Measurement For In-plane Speckle Fringes

\[ U \equiv u_x = \frac{N \lambda L}{r_x} \]
\[ V \equiv u_y = \frac{N' \lambda L}{r_y} \]

\( \bar{f}(r_x, r_y) \) = spatial frequency indicator

\( \lambda \) : wavelength of laser light

\( N, N' = 0, 1, 2, 3 \ldots \) fringe nodes

Displacement Measurement by Speckle

\[ U \text{ – field} \]
\[ V \text{ – field} \]

Moiré Fringes  Speckle Fringes
**Projection Speckle Method for Mapping Depth or Deflection**

\[ w = \frac{N\lambda L}{mr \tan \theta} \]

- \( m \) = magnification
- \( \lambda \) = wavelength
- \( N = 0,1,2,3... \)
- \( r \) = Fourier Filtering distance

**Shape Measurement by Laser Speckle**

G. K. Jaisingh and F.P. Chiang, 1 October 1981/ Vol. 20, No. 19/ App. Optics

A technique is presented to determine surface contours of 3-D objects by laser speckle interferometry. A double exposure specklegram is recorded by giving the object a small tilt between exposures. Surface contours are obtained by Fourier filtering of the specklegram.
Slope Measurement by Laser Speckle or Reflection Moiré

\[
\frac{\partial w}{\partial x} = \frac{N}{2D} \frac{\lambda L}{r_x}, \quad \frac{\partial w}{\partial y} = \frac{N}{2D} \frac{\lambda L}{r_y}
\]

Thus the Reflection Moiré and Reflection Laser Speckle Methods are Equivalent

\[
P \text{ (moiré)} = \frac{N\lambda}{r} \text{ (speckle)}
\]

Slope Fringe by Moiré

Slope Fringe by Laser Speckle
Versatilities of Speckle Method I

Displacement or slope contours along different directions

Partial slope contours of a clamped centrally loaded triangular plate. Different patterns are obtained by placing the filtering aperture at different angles at the transform plane as shown at top figure.

Versatilities of Speckle Method II

Displacement or slope contours with Different sensitivity

Fringe patterns of partial slope contours with different sensitivity of a clamped circular plate under concentrated central load.

Circles at top of figure depict the position of filtering aperture at transform plane.
Versatilities of Speckle Methods III

Time Average Fringes

with different sensitivity

with different orientations

Projection Grating Method for Measuring Shape

Phase-shifting Method

Huang et al

2-D photo

$I_1 (-3p/2)$

$I_2 (0)$

$I_3 (3p/2)$

Wrapped phase map

3-D model
A Piece of Rabbit Hair of above 20 micron as Mapped by the Digital Projection Grating Technique

An optical method of generating slope and curvature contours of bent plates

Optical arrangement of the method

\[
\Delta \xi = \xi - \xi_0 = f \tan (\alpha_i + 2\beta\Phi_i) - f \tan \alpha_i = f \frac{\tan \Phi_i \sec^2 \alpha_i}{1 - \tan \alpha_i \tan 2\Phi_i}
\]

(1)

\[
\Delta \xi = \frac{2f\Phi_i}{\cos^2 \alpha_i} \quad \text{or} \quad \Phi_i = \frac{\Delta \xi \cos^2 \alpha_i}{2f}
\]

(2)

\[
\frac{\partial W}{\partial x} = \frac{(n - n_0)p}{2f} \cos^2 \alpha_i \quad \text{or} \quad \frac{\partial W}{\partial x} = \frac{Np \cos^2 \alpha_i}{2f}
\]

(3)
Image shifting due to diffraction

\[ \Delta u = z \tan (\beta + \theta) - \tan \beta = z \cdot \sec^2 \beta \tan \theta \tan \beta \tan \theta \]  

(4)

\[ \Delta u = \frac{\lambda z}{p \cos^3 \beta} \]  

(5)

\[ I_0 (u) = \frac{1}{2} \left[ 1 + \cos 2n (u + \pi) \right] \]  

(6)

\[ I_1 (u) = \frac{1}{2} \gamma_1 \left[ 1 + \cos 2n (u + \Delta u + \pi) \right] \]  

(7)

\[ I_{-1} (u) = \frac{1}{2} \gamma_1 \left[ 1 + \cos 2n (u - \Delta u + \pi) \right] \]  

(8)
\[ I_r(u) = \frac{1}{2} \left[ 1 + 2\gamma_1 + \cos 2n\pi + \gamma_1 \left[ \cos 2(n + \Delta n)\pi + \cos 2(n - \Delta n)\pi \right] \right] \]
\[ = \frac{1}{2} \left[ 1 + 2\gamma_1 + \cos 2n\pi \left[ 1 + 2\gamma_1 \cos (\Delta n)\pi \right] \right] \]  \hspace{1cm} (9)

Assume \( \gamma_1 = \frac{1}{2} \)

\[ I_r(u) = \frac{1}{2} \left[ 2 + \cos 2n\pi \left[ 1 + \cos (\Delta n)\pi \right] \right] \]  \hspace{1cm} (10)

For \( \Delta n = 0, \pm 1, \pm 2, \ldots \), Eq. (10) reduces to

\[ I_r(u) = 1 + \cos 2n\pi \]  \hspace{1cm} (11)

\[ \frac{\Delta \Phi_s}{\Delta u} = \frac{\Phi_s(u + \Delta u) - \Phi_s(u)}{\Delta u} = \frac{n(u + \Delta u) - n(u)}{\Delta u} \frac{p \cos^2 \alpha_s}{2f} \]
\[ = \frac{\Delta n p^2 \cos^2 \alpha_s \cos^3 \beta}{2f \lambda} \]  \hspace{1cm} (12)

Noting that \( z = (b - 1)f \) And the magnification \( M = \frac{u}{x} = \frac{b}{a} = (b - 1) \)

\[ \frac{\partial^2 W}{\partial x^2} \approx \frac{\Delta \Phi_s}{\Delta x} = (b - 1) \frac{\Delta \Phi_s}{\Delta u} \]  \hspace{1cm} (13)

Eq. 12 is reduced to

\[ \frac{\partial^2 W}{\partial x^2} \approx \frac{N' p^2 \cos^2 \alpha_s \cos^3 \beta}{2f^2} \]  \hspace{1cm} (14)

The new order of the enveloping fringes
If the field lens has a long focal length and small aperture the angle $\beta$ may be approximated by $\alpha_x$

$$\frac{\partial^2 w}{\partial x^2} \approx \frac{N' p^2 \cos^5 \alpha_x}{24f^2} \quad (15)$$

$$\frac{\partial^2 w}{\partial x \partial y} \approx \csc 2\delta \left( \frac{\partial^2 w}{\partial \delta^2} - \frac{\partial^2 w}{\partial x^2} \cos^2 \delta - \frac{\partial^2 w}{\partial y^2} \sin^2 \delta \right) \quad (16)$$

$$\frac{\partial w}{\partial x} = \frac{Np}{2f} \quad \text{and} \quad \frac{\partial^2 w}{\partial x^2} = \frac{N' p}{24f^2} \quad (17)$$

Slope contours of a cantilever beam under tip load and comparison between theory and experiment
Slope contours of a clamped circular plate under centrally applied concentrated load and comparison between theory and experiment

Curvature contours of a cantilever beam under tip load and comparison between theory and experiment
Curvature contours of a clamped circular plate under centrally applied concentrated load and comparison between theory and experiment

Slope and curvature contours along x and y directions of a clamped triangular plate under uniform pressure
Propagating slope contours of a square cantilever beam under impact (numbers in $\mu$sec)

Propagating curvature contours of a square cantilever beam under impact (numbers in $\mu$sec)
Partial List of Relevant References

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THANK YOU
for
Your Attention